Covariant two-point function for linear gravity in de Sitter space

Mohammad Vahid TAKOOK *
Departement of physics, Razi University, Kermanshah, IRAN

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Abstract

The Wightman two-point function for the gravitational field in the linear approximation (the rank-2 "massless" tensor field) on de Sitter space has a pathological behaviour for large separated points (infrared divergence). This behaviour can be eliminated in the two-point function for the traceless part of this field if one chooses the Gupta-Bleuler vacuum. But it is not possible to do the same for the pure trace part (conformal sector). We briefly discuss the consequences of this pure trace behaviour for inflationary models.

1 Introduction

The graviton propagator on de Sitter (dS) space (in its usual linear approximation for the gravitational fields) for large separated points has a pathological behaviour (infrared divergence) [Allen, Turyn, 1987; Floratos, Iliopoulos, Tomaras, 1986; Antoniadis, Mottola, 1991]. Some authors proposed that infrared divergence could rather be exploited in order to create instability of the dS universe [Ford, 1985; Antoniadis, Iliopoulos, Tomaras, 1986]. The field operator for linear gravity in dS space has been considered in this way by Tsamis and Woodard in terms of flat coordinates which cover only one-half of the dS hyperboloid [Tsamis, Woodard, 1992]. They have examined the possibility of quantum instability and they have found a quantum field which breaks dS invariance. However, we show that this behaviour problem for the traceless part of the field disappears if one uses the Gupta-Bleuler vacuum defined by [de Bievre, Renaud, 1998; Gazeau, Renaud, Takook, 1999]. On the other hand, such a procedure is unsuccessful for the pure-trace part of the field (conformal sector). In the general relativity framework, one cannot associate a dynamics to the conformal sector because the physical content of this field is not apparent. It is coordinate or gauge dependent. Therefore one may think that its behaviour troublesome may originate from imposing the gauge invariance and has no actual physical consequence. But, in the presence of a matter quantum field, that part of the metric acquires a dynamical content and the problem appears in any attempt to quantize it.

*e-mail: takook@ccr.jussieu.fr

In a previous paper, we have shown that one can write the rank-2 "massive" tensor field (divergencelesse or "transverse" and traceless) in terms of a projection operator and a scalar field. At the "massless" limit, there appears a singularity in the projection operator. This type of singularity appears precisely because of the divergenceless condition. By dropping the divergenceless condition, we can make the mentioned singularity in the tensor field (for its traceless part only) disappear. In quantizing this field, there appears another singularity in the Wightman two point function like in the case of the "massless" minimally coupled scalar fields [Allen, Folacci, 1987]. The latter type of singularity appears because of the zero mode problem for the Laplace-Beltrami operator on dS space. In order to solve it, we must follow the procedure already used for a completely covariant quantization of the minimally coupled scalar field [Gazeau, Renaud, Takook, 1999].

The organization of this paper is the following. Section 2 is devoted to the traceless field and it is explained how the choice of the Gupta-Bleuler vacuum eliminates pathological behaviour. In Section 3 we examine the questions raised by the pure-trace part. Section 4 is a brief conclusion on the inflationary universe scenario.

2 Traceless part

Here, we briefly recall our de Sitterian notations. The de Sitter space-time is made identical to the four dimensional one-sheeted hyperboloid

$$X_H = \{x \in \mathbb{R}^5; x^2 = \eta_{\alpha\beta} x^\alpha x^\beta = -H^{-2}\}, \quad \alpha, \beta = 0, 1, 2, 3, 4, \tag{1}$$

where $\eta_{\alpha\beta}$ =diag$(1, -1, -1, -1, -1)$. The de Sitter metrics is

$$ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta = g^{dS}_{\mu\nu} dX^\mu dX^\nu, \quad \mu = 0, 1, 2, 3, \tag{2}$$

where X^μ are the 4 space-time coordinates in dS hyperboloid. We use the tensor field notation $K_{\alpha\beta}(x)$ with respect to the ambiant space, and the transversality condition $x.K(x) = 0$ is imposed. In this notation, it is simpler to express the tensor field (and also the two-point function) in terms of scalar fields.

The two-point function for the "massive" spin-2 field $K^{tt}_{\alpha\beta}(x)$ ("transverse" or divergenceless and traceless) is defined by [Gazeau, Takook]

$$\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x') = \langle \Omega, K^{tt}_{\alpha\beta}(x) K^{tt}_{\alpha'\beta'}(x')\Omega\rangle$$
$$\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x') = D^{tt}_{\alpha\beta\alpha'\beta'}(x, x')\mathcal{W}(x, x'). \tag{3}$$

$\mathcal{W}(x, x')$ is the Wightman two-point function for the massive scalar field on dS space.
$D^{tt}_{\alpha\beta\alpha'\beta'}(x, x')$ is a projection tensor, which satisfies the "divergencelesse" and traceless conditions. In the limit of the "massless" spin-2 field there appear two types of singularity in the two-point function. The first one lies in the projection tensor $D^{tt}_{\alpha\beta\alpha'\beta'}(x, x')$ and it disappears if one fixes the gauge (the dropping of the divergenceless condition). The other one lies in the scalar Wightman two-point function $\mathcal{W}(x, x')$ (the minimally coupled scalar field) and it disappears if we follow the procedure presented in [Gazeau, Renaud, Takook, 1999]. Then the two-point function is defined by [Gazeau, Renaud, Takook]

$$\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x') = \langle \Omega, K^{t}_{\alpha\beta}(x) K^{t}_{\alpha'\beta'}(x')\Omega\rangle$$

$$\mathcal{W}_{\alpha\beta\alpha'\beta'}(x,x') = \Delta^t_{\alpha\beta\alpha'\beta'}(x,\partial;x',\partial')\mathcal{W}(x,x'), \tag{4}$$

where $\Delta^t(x,\partial;x',\partial')$ is a projection tensor which satisfies the traceless condition. $\mathcal{W}$ is the two-point function for the minimally coupled scalar field in the Gupta-Bleuler vacuum [Takook, 1997]

$$\mathcal{W}(x,x') = \frac{iH^2}{4\pi}\epsilon(x^0 - x'^0)[\delta(1-\mathcal{Z}(x,x')) - \theta(\mathcal{Z}(x,x')-1)], \tag{5}$$

where $\mathcal{Z} = -H^2 x.x'$ and $\epsilon(x^0 - x'^0) = \begin{cases} 1 & x^0 > x'^0 \\ 0 & x^0 = x'^0 \\ -1 & x^0 < x'^0. \end{cases}$

3 Conformal sector

The tensor field that we considered in the previous section is traceless. But in the general case the tensor field consists of a traceless part and a pure-trace part (conformal sector):

$$K_{\alpha\beta}(x) = K^t_{\alpha\beta}(x) + K^{pt}_{\alpha\beta}(x). \tag{6}$$

The pure trace part can be written in the form

$$K^{pt}_{\alpha\beta}(x) = \frac{1}{4}\theta_{\alpha\beta}\psi,$$

where ψ is scalar field and $\theta_{\alpha\beta} = \eta_{\alpha\beta} + H^2 x_\alpha x_\beta$. With a certain choice in the gauge condition , we are able to write down the following field equation for the scalar field ψ [Gazeau, Renaud, Takook]

$$(\Box_H - 5H^2)\psi = 0. \tag{7}$$

So this field cannot be interpreted in terms of a unitary irreducible representation of the dS group. Difficulties arise when we want to quantize such fields which show negative squared mass in their wave equation. The corresponding two-point functions have a pathological large-distance behaviour (infrared divergence)[Gazeau, Renaud, Takook]. We just emphasize on the fact that, so far, this degree of freedom should not appear as a physical one.

4 Conclusion

We conclude that the pathological large-distance behaviour for the physical degree of freedom of the linear gravity in the Wightman two-point function can be easily cured. Antoniadis, Iliopoulos and Tomaras have also shown that the pathological large-distance behaviour of the graviton propagator on a dS background does not manifest itself in the quadratic part of the effective action in the one-loop approximation [Antoniadis, Iliopoulos and Tomaras; 1996]. That means that this behaviour may be gauge dependent and it should not appear in an effective way in a physical quantity. On the other hand, it exists in an irreducible way in the pure-trace part (conformal sector). The conformal sector may be interesting for inflationary universe scenarii. In these theories, one introduces an inflaton scalar field. Because of this field, the conformal sector of the metric becomes dynamical and it must be quantized [Antoniadis, Mazure, Mottola,

1997]. Then it produces a gravitational instability. This gravitational instability and the primordial quantum fluctuation of the inflaton scalar field define the inflationary model. The latter can explain the formation of the galaxies, clusters of galaxies and the large scale structure of the universe [Lesgourgues, Polarski, Starobinsky, 1998].

We may conclude that the quantum instability of dS space and the breaking of the dS invariance are both due to the quantization of the conformal sector.

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